ON THE STRUCTURE OF $2 \times \infty$ BIVARIATE DISTRIBUTIONS WHICH ARE TOTALLY POSITIVE OF ORDER TWO

BY K. SUBRAMANYAM[1] AND M. BHASKARA RAO[1]

University of North Carolina at Wilmington and North Dakota State University

Let X and Y be two random variables such that X takes only two values 1 and 2. The notion of total positivity of order two for the joint probability distribution of X and Y is discussed in this paper from the viewpoint of convex analysis. The set of all $2 \times \infty$ probability measures which are totally positive of order two and with fixed second marginal probability measure is shown to be convex. Some of the extreme points of this set are explicitly spelled out, and an integral representation theorem in terms of extreme points is presented in a special case.

1. Introduction. Let X and Y be two random variables having a joint probability density function $f(\cdot, \cdot)$ with respect to some product probability measure λ on the Borel σ-field of R^2. The random variables X and Y are said to be totally positive of order two if the determinants

$$\begin{vmatrix} f(x,y) & f(x,y') \\ f(x',y) & f(x',y') \end{vmatrix}$$

are nonnegative for $-\infty < x \leq x' < \infty$ and $-\infty < y \leq y' < \infty$ a.e. $[\lambda]$. See Karlin (1968, p. 12). For its relation with other notions of dependence and further ramifications, see Barlow and Proschan (1981). See also Lehmann (1966).

The main purpose of this article is to perform extreme point analysis on the notion of total positivity of order two. What this means is that we look at the set of all bivariate probability density functions, examine convexity of this set, and if convex, enumerate all its extreme points. This kind of analysis was carried out on a limited scale in Subramanyam and Bhaskara Rao (1988). It was shown that the set of all bivariate probability density functions which are totally positive of order

[1]Supported by AFOSR contract AFSO-88-0030.

AMS 1980 subject classifications. Primary 60E05; secondary 62H05.

Key words and phrases. Bivariate distribution, total positivity of order two, convex set, compact set, extreme points.

The authors are very much grateful to the referee for many of his perceptive comments which led to a substantial improvement in the presentation of the paper.