MULTIVARIATE DISTRIBUTIONS GENERATED FROM MIXTURES OF CONVOLUTION AND PRODUCT FAMILIES

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A number of standard univariate distributions can be represented as mixtures of other standard distributions. In this paper such mixture representations are exploited to generate families of multivariate distributions with given marginals. Attention is confined to mixtures of parametric families where the parameter appears as the order of a convolution or as a power of the distribution or survival function. The mixture structure yields properties of the generated multivariate distributions such as total positivity, association and infinite divisibility. Examples obtained include the bivariate Poisson, binomial, negative binomial, normal, chi-square, logistic and Pareto distributions.

1. Introduction. For any given parametric family of distributions $F(\cdot \mid \theta)$, it is possible to regard the parameter θ as the value of a random variable Θ with distribution G, say. Then $F(\cdot \mid \theta)$ is a conditional distribution given $\Theta = \theta$ and the corresponding unconditional distribution

$$H(x) = \int F(x \mid \theta) dG(\theta) \tag{1}$$

is a mixture.

Here, both x and θ can be vectors, often of different dimensions. Many examples arise in which θ is a scalar and $F(\cdot \mid \theta)$ is the product of its marginals. Then (1) takes the form

$$H(\mathbf{x}) = \int \Pi F_i(x_i \mid \theta) dG(\theta), \tag{2}$$

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