POSITIVE DEPENDENCE CONCEPTS FOR ORDINAL CONTINGENCY TABLES

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This paper provides a connection for cross-classified variables between ordinal contingency tables and positive dependence concepts. Various bivariate positive dependence properties are reinterpretted in terms of odd ratios for certain subtables of the two-way array.

1. Introduction. The primary purpose of this paper is to provide a bridge between two literatures: ordinal contingency tables and positive dependence concepts. Both focus on relationships among discrete random variables. Contingency table research has concentrated on statistical analysis of parametric models for cell frequencies or probabilities. Well known examples of such models are Bishop, Fienberg, and Holland's (1975) log-linear models, and Goodman's (1979, 1985) association models. In the modeling of ordinal contingency tables, the ordered structure of the levels of the cross-classifying variables is usually translated into a related ordering constraint on the corresponding model parameters, e.g., Douglas and Fienberg (1990).

The dependence literature, on the other hand, has focused on probabilistic characterizations, which lead to the study of properties associated with the cell probabilities, rather than with the parameters in models. For a discussion of positive dependence properties and their interrelationships, see Barlow and Proschan (1981).

In this paper, we study in depth the connection between ordinal contingency table parametrizations and positive dependence properties of cross-classified variables. The relationships we examine are established through generalized odds ratios in a contingency table. A few of these connections have been considered previously by Agresti (1984), Grove (1984), and Yanagimoto (1972). Throughout our presentation, we pursue the dual goals of promoting potentially new models for contingency table researchers and bringing ordinal statistical models and techniques to researchers in positive dependence.