CONVEX-ORDERING AMONG FUNCTIONS, WITH APPLICATIONS TO RELIABILITY AND MATHEMATICAL STATISTICS

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Hardy, Littlewood, and Pólya (1952) introduced the notion of one function being convex with respect to a second function and developed some inequalities concerning the means of the functions. We use this notion to establish a partial order called convex-ordering among functions. In particular, the distribution functions encountered in many parametric families in reliability theory are convex-ordered. We have formulated some inequalities which can be used for testing whether a sample comes from F or G, when F and G are within the same convex-ordered family. Performance characteristics of different coherent structures can also be compared with respect to this partial ordering. For example, we will show that the reliability of a $k + 1$-*out-of-n* system is convex with respect to the reliability of a k-*out-of-n* system.

When F is convex with respect to G, the tail of the distribution F is heavier than that of G; therefore, our convex-ordering implies stochastic ordering. Convex-ordering is also related to total positivity and monotone likelihood ratio families. This provides us a tool to obtain some useful results in reliability and mathematical statistics.

1. Introduction. Notions of partial ordering among survival distributions have played a useful role in providing numerous inequalities in reliability. The notion of a random variable X with distribution F being stochastically larger than another random variable Y with distribution G is well known in the literature. Van

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