STOCHASTICALLY MONOTONE DEPENDENCE

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The notion of monotone dependence, which has played a key role in reliability theory, is generalized to that of "stochastically monotone dependence." The idea here is that since two lifelengths are dependent or independent based on the disposition of a conditioning variable, they are unconditionally stochastically dependent or independent. A measure of stochastic dependence is introduced and the measure used for comparing the correlations of pairs of random variables which can now be described as being "highly stochastically correlated" or "weakly stochastically correlated." Extensions to the multivariate case are possible and the ideas illustrated via examples. This paper is expository; its purpose is to propose a natural idea and to explore its ramifications.

1. Introduction and Motivation.

An important, though little noticed, principle of probability theory is that the notions of dependence and independence are conditional, the conditioning being done on some observable or unobservable quantity, say Θ. It is common to think of Θ as a "parameter" and this is the point of view that we adopt. A consequence of the above is that unconditionally the notions of dependence and independence must be stochastic. That is, one should not make an unqualified judgment that lifelengths X_1 and X_2 are dependent or independent—rather one may talk in terms of the *probability that they are dependent or independent*. This is contrary to current thinking although the literature on artificial intelligence [cf. Pearl (1989)] appears to be taking cognizance of this fact. In this paper, we explore the ramifications of the above formulation, and in the sequel raise questions pertaining to the everyday used notions of covariance and correlation.

By way of some motivation, consider a system of two components with lifelengths X_1 and X_2, operating in an environment which is characterized by an