

INTRODUCTION

Concepts of positive dependence are becoming increasingly important in probability, statistics and their applications. While these concepts are traditionally viewed as focusing on positive and negative dependence for random vectors, they also are related to broader issues in the modeling and the analysis of multivariate data, and, in particular, ordinal data.

Historically, positive dependence for the multivariate normal distribution had been synonymous with positive correlations. Other subsequently developed multivariate distributions were often interpreted with this perspective. It was eventually realized that positive correlations can have substantially different meanings for other multivariate distributions than they have for the multivariate normal. In fact, it has been more recently demonstrated that several different important positive dependence concepts which are equivalent for multivariate normal distributions are not equivalent, in general, for multivariate distributions. In particular the concept of association is stronger than positive orthant dependence which is stronger than positive correlations. Thus, in a certain sense, many of the positive dependence concepts discussed or referenced in this volume, are outgrowths of original attempts to nonparametrically capture and extend certain properties of the multivariate normal. Additionally, other types of dependence came about from extending certain univariate properties, such as the memoryless property of the exponential distribution.

From the point of view expressed above, the theoretical origins of this research area include the fundamental works of Lehmann (1966) concerning orthant dependence, Esary, Proschan and Walkup (1967) dealing with the concept of association, and Marshall and Olkin (1967) modeling multivariate distributions. These three papers drew upon a rich historical stream and, in turn, have spawned numerous applications and inspired other related dependence concepts. The sources of this historical stream range broadly from reliability and mathematical inequalities to nonparametric statistical modeling and measures of association. Some of the researchers involved in these pioneering efforts include Goodman, Hardy-Littlewood-Polya, Hoeffding, Karlin, Kendall, Kruskal, Lancaster, Šidák, Sobel, and Tukey.

Barlow and Proschan (1981) further developed positive dependence concepts in their book (first printed in 1975) particularly in the bivariate case, and Tong (1980) provides additional material and development. A parallel development occurred independently in the mathematical physics literature (see Fortuin, Kastelyn and Ginibre (1971)). For a long time it was felt that negative dependence concepts were the mirror image of positive dependence. That this was not the case was demonstrated by Karlin and Rinott (1980), Block, Savits, and Shaked (1982), and by Joag-dev and Proschan (1983) among others. Related review articles on inequalities and dependence are those by Eaton (1982) and by Block and Sampson