Institute of Mathematical Statistics

LECTURE NOTES — MONOGRAPH SERIES

PARTIAL LIKELIHOOD AND ESTIMATING EQUATIONS

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Abstract

Consider a regression model for discrete-time stochastic processes, with a (partially specified) model for the conditional distribution of the response given the covariate and the past observations. Suppose we also have some knowledge about how the parameter of interest affects the conditional distribution of the covariate given the past. We assume that these two model assumptions give rise to two martingale estimating functions, and determine an optimal combination. We indicate for the case of jump processes how our result carries over to continuous time. The resulting estimators are efficient.

1 Introduction

Suppose we know something about how the parameter of interest in a regression model appears both in the conditional distribution of the response given the covariate, and in the distribution of the covariate. How can we exploit this knowledge? Let us illustrate our approach in the case of independent and identically distributed observations (X_i, Y_i), with X_i the covariate and Y_i the response. In a regression model one usually specifies the conditional distribution of Y given X, either fully, by a parametric model, or partially. An example of a partial specification is a model for the conditional mean of Y given X, say $\mathrm{E}(Y|X) = \vartheta X$. More generally, we specify a function $\overline{g}_\vartheta(X,Y)$ such that $\mathrm{E}(\overline{g}_\vartheta(X,Y)|X) = 0$. In the example, $\overline{g}_\vartheta(X,Y) = Y - \vartheta X$. We assume a similar partial specification of the distribution of the covariate X, say $\mathrm{E}g_{*\vartheta}(X) = 0$. The two functions $\overline{g}_\vartheta$ and $g_{*\vartheta}$ give rise to two estimating equations

$$\sum_{i=1}^{n} \overline{g}_\vartheta(X_i, Y_i) = 0, \quad \sum_{i=1}^{n} g_{*\vartheta}(X_i) = 0.$$

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