RESULTS AND PROBLEMS IN GAMES OF TIMING

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Abstract. This paper presents the history of investigations concerning a subclass of zero-sum two-person games called duels, which were initiated by David Blackwell and other mathematicians in the reports of the RAND Corporation in 1948-52. The second part of the paper discusses mutual relationships between discrete and non-discrete duels, and gives a review of recent more general results. The paper also discusses some open problems in the general theory and makes hypothesis on them (strongly suggested by previous results).

1. INTRODUCTION

In 1948 the RAND Corporation collected a team of mathematicians, statisticians, economists, and social scientists to analyze "the uncertainties" in the global world situation and to construct a blue-print for the optimal operation against that. One of the results of this study, achieved within a component of the program, was the solution of many problems formulated in the form of some zero-sum two-person games, called duels, or games of timing when considered in a more general sense. Particularly, David Blackwell, one of the members of that team, was very instrumental in formulating and solving several versions of such games. He, together with M. Shiffman, M. A. Girshick, L. S. Shapley, R. Bellman, I. Glicksberg and others, initiated a new topic within zero-sum games at that time, and recognized the wide scope of possible applications of games of timing, particularly in the description and explaining of some conflict situations in economics. Since then many new general problems in games of timing have been formulated and many important and interesting results have been achieved. However, to say more in detail about it, at first, we must give a definition of games of timing in a sufficiently general form to include the whole rich collection of all different duel-models studied in the literature. We will do it in a slightly different (but equivalent) convention in comparison to that adopted by the pioneers of this topic mentioned above.

Consider the following model of a zero-sum game: There are two Players 1 and 2 with initial amounts M_1 and M_2 of some homogeneous resources, respectively. It is assumed that they should distribute some or all of their

¹Research supported by Grant KBN grant 2 P03A 007 10/96.

Key words and phrases. Duel, noisy, silent, discrete duel, non-discrete duel, game of timing. AMS subject classifications. 90D, 55, 60, 05