Bayesian Robustness
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LOCAL ROBUSTNESS OF BAYES FACTORS FOR NONPARAMETRIC ALTERNATIVES

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In this paper we consider a particular Bayes factor B for comparing a fixed parametric model against a nonparametric alternative, and we investigate its local sensitivity to the sampling distribution. The nonparametric alternative is constructed by embedding the parametric model, characterized by a d.f. F_0 known up to a real parameter θ, into a mixture of Dirichlet processes. More precisely, conditionally on θ, F_0 represents the mean of a random d.f. which is assumed to be a Dirichlet Process. So, for the Bayes factor B, sensitivity to perturbations of the sampling distribution F_0 and sensitivity to small departures from the fixed Dirichlet process parameter are the same problem. Here we consider B as a (non ratio-linear) functional defined on a set of sampling d.f.'s and maximize its first von Mises derivative over this set. In particular, mixture and density bounded sets are considered.

1. Introduction. A Bayesian analysis may depend critically on the modeling assumptions which include prior, sampling distribution and loss function. Therefore, it is useful to assess the sensitivity of inferences to modest changes in the specification of the problem by means of a so called robustness analysis. On this subject there exists an extensive literature. A general discussion and comprehensive lists of references can be found in Berger (1984), (1990), (1994), Gustafson, Wasserman and Srinivasan (1994) and Wasserman (1992). Most of the literature is concerned with global sensitivity to prior specification and focusses on posterior expectations as inferences of interest. In this article we discuss *local* sensitivity to the *sampling distribution* of a particular *Bayes factor* [Carota and Parmigiani (1994)].

In general, a sensitivity analysis is performed when there is uncertainty about modeling assumptions. Such uncertainty is expressed by specifying a class of inputs (for example, a class of priors or a class of sampling distributions) instead of a single one. The local sensitivity analysis examines the rate at which the inference changes relative to small perturbations to a base input in direction of the other elements in the class. It is preferred to the global analysis when the given class of inputs contains a natural reference point, either because of a very high degree of belief or because of mathematical appeal. Sometimes a local analysis can be used to construct quite accurate global robustness bounds, when exact computations are too difficult or too time consuming.

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