ON THE ROBUSTNESS OF THE INTRINSIC BAYES FACTOR FOR NESTED MODELS [1]

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Bayesian model comparisons are known to be undetermined when improper priors are employed. The Intrinsic Bayes factor (IBF) is a general automatic procedure for model comparison proposed in Berger and Pericchi (1993) which addresses the difficulties that arise when improper priors are employed. An appealing justification of the IBF is that it asymptotically corresponds to actual Bayes factors of particular priors. Such priors are called instrinsic priors and can be obtained as solutions of two functional equations. In this paper we consider issues related to the robustness of the IBF in the nested model situation.

1. Introduction. The problem of comparing two models is addressed, from a Bayesian perspective, in the following way: consider a set of data X that have density $f_i(\mathbf{x}|\theta_i)$ under model $M_i, i = 1, 2, \theta_i \in \Re^{k_i}$, and suppose that prior distirbutions $\pi_i(\theta_i)$ are selected for the parameters of each model. Select prior probabilities for each model and update them using the Bayes factor defined as

$$B_{21} = \frac{\int f_2(\mathbf{x}|\theta_2)\pi_2(\theta_2)d\theta_2}{\int f_1(\mathbf{x}|\theta_1)\pi_1(\theta_1)d\theta_1} \quad . \tag{1}$$

The same tool is not available for comparing models with improper priors, since these are defined only up to a multiplicative constant, leaving the Bayes factor undetermined. Berger and Pericchi (1995) introduced the idea of *intrinsic Bayes factor* (IBF) to address this problem. The method consists of using a subsample of minimal size to obtain a proper posterior distribution that is used as a proper prior to compute a Bayes factor for the remainder of the data, the results are averaged over all possible training samples to produce the IBF. More precisely, letting L be the number of all training samples, the arithmetic version of the IBF is defined as

$$B_{21}^{AI} = B_{21}^{N}\frac{1}{L}\sum_{l=1}^{L} B_{12}^{N}(\mathbf{x}(l)) \tag{2}$$

where $B_{21}^N = m_2^N(\mathbf{x})/m_1^N(\mathbf{x})$; $B_{12}^N(\mathbf{x}(l)) = m_1^N(\mathbf{x}(l))/m_2^N(\mathbf{x}(l))$,

$$m_i^N(t) = \int f_i(t|\theta_i)\pi_i^N(\theta_i)d\theta_i \quad ,$$

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