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# LOCAL ROBUSTNESS AND INFLUENCE FOR CONTAMINATION CLASSES OF PRIOR DISTRIBUTIONS

BY ELÍAS MORENO, CARMEN MARTÍNEZ AND J. ANTONIO CANO
*University of Granada and University of Murcia*

In the well-known $\varepsilon$-contamination class of prior distributions $\Gamma = \{\pi(\theta) : \pi(\theta) = (1-\varepsilon)\pi_0(\theta) + \varepsilon q(\theta),\ q \in Q\}$ , $\varepsilon$ represents the degree of uncertainty on the base prior $\pi_0(\theta)$ and $Q$ the allowed class of contaminations. We argue here that the uncertainty we have on $\pi_0(\theta)$ typically is stronger on its tails than on its body. This idea is formalized through a more general $\varepsilon(\theta)$-contamination class that might be seen as a local robustification of $\pi_0(\theta)$. When $Q$ is defined by quantile constraints, the admissible classes of functions $\varepsilon(\theta)$ capable of maintaining the prior information for the resulting priors $\pi(\theta)$ are characterized and robust posterior analysis is carried out. Influence analysis is also considered. In this setting Fréchet derivatives are useful tools: they are easily interpreted and easily computed. Interactive robustness based on influence analysis is discussed.

**1. Introduction.** Let $\mathbf{x}$ be a set of data which will be assumed to arise from a density $f(\mathbf{x} \mid \theta)$, where $\theta$ denotes unknown parameters in the space $\Theta$. Robust Bayesian Analysis assumes uncertainty on the prior distribution $\pi(\theta)$ and models such an uncertainty by considering classes of priors for which robustness analyses are carried out. One of the most interesting classes is the contamination class

$$(1) \qquad \Gamma = \{\pi(\theta) : \pi(\theta) = (1-\varepsilon)\pi_0(\theta) + \varepsilon q(\theta),\ q \in Q\},$$

which is proposed as follows. Some prior beliefs are established and a base prior $\pi_0(\theta)$ matching these requirements is elicited. A constant $\varepsilon$, $0 < \varepsilon < 1$, reflecting our degree of uncertainty on the functional form of $\pi_0(\theta)$, is specified. Finally, the class $Q$ of all possible priors compatible with the prior beliefs is considered.

Prior beliefs are expressed by the probabilities of some sets $C_i$, $i \geq 1$, which form a partition of the parameter space $\Theta$. Therefore, the prior should be any probability measure $\pi(\theta)$ such that $P^{\pi}(C_i) = \alpha_i$, $i \geq 1$, where $\alpha_i$ are known. The base prior $\pi_0(\theta)$ is then chosen such that $P^{\pi_0}(C_i) = \alpha_i$, $i \geq 1$, and the class $Q$ is

$$(2) \qquad Q = \{q(\theta) : P^{q}(C_i) = P^{\pi_0}(C_i),\ i \geq 1\}.$$

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