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LOCAL SENSITIVITY ANALYSIS IN BAYESIAN DECISION THEORY [1]

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We study local sensitivity in Bayesian Decision Theory, allowing for imprecision in the Decision Maker's preferences and beliefs. Our tools are based on Fréchet derivatives of operators and their norms. They allow us to detect cases in which robustness is lacking and, eventually, the most critical judgments determining choice.

1. Introduction. Our initial framework is that of Bayesian Decision Theory and Inference, see Savage (1972). Several authors suggest that Bayesian foundations place excessive demands on the Decision Maker's (DM) judgments. This motivates the development of tools to check the sensitivity of conclusions of a Bayesian analysis with respect to changes in the inputs. Berger (1994) provides an excellent review. However, most work has concentrated on sensitivity to the prior. As we suggest in our discussion to that paper, fundamental (Ríos Insua and Martín, 1995) and practical (Ríos Insua and Martín, 1994) issues suggest developing a general framework for sensitivity analysis allowing for perturbations both in preferences and beliefs.

This paper studies this question from a local perspective, that is, we study whether small perturbations in the inputs to the analysis lead to important changes in the conclusions. For this, we use Fréchet derivatives. We generalize previous results by, among others, Diaconis and Freedman (1986), Srinivasan and Trusczynska (1995), Basu et al. (1993), Sivaganesan (1993) and, specially, Ruggeri and Wasserman (1993), to the more difficult case, see Berger (1994), of imprecision in both the utility and the prior.

We think of our results in an iterative fashion. Our analyses allow us to detect cases in which robustness is lacking. Moreover, we provide procedures suggesting what additional information we must elicit from the DM to increase robustness. It is specially relevant that this information is meaningful to the DM, against somewhat skeptical opinions (Berger, 1994; Das Gupta, 1995). Hence, we could think of incorporating our results into a more general framework for sensitivity analysis in Decision Theory, see Ríos Insua (1990).

We shall use the following notation: S will designate the set of states s, endowed with a σ-field $\mathcal{B}$. P will be the prior distribution modeling the DM's beliefs, updated to the posterior $P(\cdot|x)$, when x is the result of an experiment with likelihood $l(x|s)$ over a sample space X. The DM makes decisions $a \in \mathcal{A}$, the space of alternatives. We associate a consequence $c \in \mathcal{C}$,

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