Iteration of IID Random Maps on R^+

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Abstract

Let $\{X_n\}$ be a Markov chain on R^+ generated by the iteration scheme $X_{n+1} = C_{n+1}X_n g(X_u)$, where $\{C_n, g_n(\cdot)\}$ are i.i.d. such that $\{C_n\}$ are nonnegative r.v. with values in $[0, L]$, $L \leq \infty$, $\{g_n\}$ are continuous functions from $[0, \infty) \to [0, 1]$ with $g_n(0) = 1$. This paper presents a survey of recent results on the existence of nontrivial stationary measures, Harris irreducibility and uniqueness of stationary measures, convergence and persistence. Four well known special cases i.e. the logistic, Ricker, Hassel and Vellekoop-Högnas models are discussed.

Keywords: Markov chains, IID random maps, Stationary measures, Harris reducibility

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1 Introduction

A topic of some interest to Professor Rabi N. Bhattacharya, whom the present volume honors, and to which he has contributed substatially is the iteration of i.i.d. random quadratic maps on the unit interval $[0, 1]$. Beginning with the paper Bhattacharya and Rao [7] where they analyzed the case of i.i.d. iteration of two quadratic maps using the Dubins-Freedman [9] results on random monotone maps on an interval, Professor Bhattacharya has obtained a number of interesting results on the uniqueness and support of the stationary distribution as well as on rates of convergence. For these the reader is referred to Bhattacharya and Majumdar [6] and Bhattacharya and Waymire [8].

In the present paper we study Markov chains generated by iteration of i.i.d. random maps on R^+ that are restricted to the class of functions $f : R^+ \to R^+$ such that they possess a finite, positive derivative at 0, vanish at 0 and have a sublinear growth for large values. This class is of relevance and use in population ecology and growth models in economics. The conditions imposed on f in this class reflects two features common in ecological modelling, namely, i) for small values of the population size X_n at time n, the population size X_{n+1} at time $n + 1$ is approximately proportional to X_n with a random proportionality constant while for large values of X_n, competition sets in and the linear growth is scaled down by a factor. This class includes many of the known models in the ecology literature such as the logistic maps, Ricker maps, Hassel maps and Vellekoop-Högnas maps, as explained in the next section.

Here is an outline of the rest of the paper. In the next section we describe the basic mathematical set up and establish some results for Feller chains on R^+. In section 3 we describe a set of necessary and two sets of sufficient conditions for the existence of stationary measures with support in $(0, \infty)$. In section 4, a trichotomy into subcritical, critical and supercritical cases is introduced and convergence results for the subcritical and critical cases are provided. Section 5

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