## ADAPTIVE TUNING, 4-D VAR AND REPRESENTERS IN RKHS

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This work was prepared for presentation at the European Centre for Medium-Range Weather Forecasts, Workshop on the Diagnosis of Data Assimilation Systems, Reading, England, November 1998. Four dimensional variational data assimilation, called 4D-Var in the atmospheric sciences literature, is a method for combining forecast, dynamical systems equations, prior information about properties of the atmosphere, and heterogeneous observations, to get an estimate of the evolving state of the atmosphere. 'Four dimensional' refers to one time and three space variables, as opposed to three space variables alone. In 4D-Var an entire trajectory in time is fit to the available information. Some of my recent research in this area is aimed at studying the use of GCV and GML methods in choosing the numerous tuning parameters in this fitting problem. We (abstractly) generalize the 'toy' weak 4D-Var model in Gong, Wahba, Johnson and Tribbia (1998) to include adaptive tuning of a variety of parameters throughout the 4D-Var variational problem, and note issues of sensitivity and identifiability. We discuss 'models' for model errors which include systematic, short memory and long memory errors. Finally we remark on the role of the theory of representers in reproducing kernel Hilbert spaces in the weak 4D-Var setting.

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## 1 Introduction

We first consider the general setup in the experiment in Gong, Wahba, Johnson and Tribbia (1998), which is a toy weak 4D-Var model (actually one time and one space variable) with five unknown smoothing, weighting and distributed parameters, which were simultaneously adaptively tuned using generalized cross validation (GCV) calculated via the randomized trace technique. In that setup 'model error' was generated as the difference between a 'nature' model and the 'computer' model, but white noise model errors were assumed in the weak 4D-Var variational problem. In this paper we then (i) review the use of model errors as dual variables, (ii) review the GCV and generalized maximum likelihood (GML) tuning methods, and pinpoint sensitivity issues as tunable parameters are sprinkled liberally throughout

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