The Almost Sure Number of Pairwise Sums for Certain Random Integer Subsets Considered by P. Erdös

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Abstract

Fix any $\lambda > 0$ and let $X_1, X_2, \ldots$ be independent and identically distributed 0–1 valued random variables such that

$$P(X_j = 1) = \min\left\{\sqrt{\frac{2\lambda}{\pi}\,\frac{\ln j}{j}}, 1\right\}.$$

Let $G_n = \sum_{j=1}^{\lfloor n/2 \rfloor} X_j X_{n-j}$. G_n is the number of times two numbers from the random set $S \equiv \{j : X_j = 1\}$ add to n. We evaluate the almost sure limits $\liminf_{n\to\infty} \frac{G_n}{EG_n} \equiv c_1(\lambda)$ and $c_2(\lambda) \equiv \limsup_{n\to\infty} \frac{G_n}{EG_n}$, showing that $0 \le c_1(\lambda) < 1 < c_2(\lambda) < \infty$.

Introduction

Around 1932 Sidon asked whether there exist positive integers $a_1 < a_2 < \ldots$ such that $f(n) > 0$ for all n sufficiently large and yet $\lim_{n\to\infty} \frac{f(n)}{n^\varepsilon} = 0$ for all $\varepsilon > 0$, where

$$f(n) = \#\{i \ge 1 : a_i + a_{j_i} = n \text{ for some } j_i \ge i\}. \tag{1}$$

Fix any $\lambda > 0$. Let $X_1, X_2, \ldots$ be independent random variables taking only values zero and one, as determined by the probabilities

$$P(X_j = 1) = \min\left\{\sqrt{\frac{2\lambda}{\pi}\,\frac{\ln j}{j}}, 1\right\} \equiv P_j. \tag{2}$$

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