Randomized Strategies and Terminal Distributions

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AMS 1991 Subject Classifications. 60G40, 93E20, 04A15, 28A12.
Key words and phrases.Measurable gambling, stochastic control, randomization, stopping time, Markov chain, the Skorohod stopping problem, analytic sets, saturated houses.

1 Introduction

A pure strategy in a Dubins and Savage (1976) gambling problem can be thought of as a sequence of gambles selected one-by-one from those available at each stage of the game. There are two natural ways to select a strategy at random. One method is to select the individual gambles at random at each stage. Another method is to choose the entire strategy at random from the set of all pure strategies. Our first result (Theorem 2.1) is that these two methods are equivalent in the context of measurable gambling theory. This result is related to similar results in game theory due to Kuhn (1953) and Aumann (1964), and in Markov decision processes due to Dynkin and Yushkevich (1979) and Feinberg (1996).

Our second theorem concerns the set of all possible terminal distributions that can be obtained by stopping a given Markov chain at random. Again we consider two ways to randomize - first by choosing a distribution at random from those terminal distributions that can be obtained using a nonrandom stopping time, and second by using a randomized stopping time. It turns out that the two methods lead to the same collection of terminal distributions (Theorem 4.4).

In the final section of the paper, we consider a third way to obtain a randomized terminal distribution. Namely, the decision to stop is made at random at each stage. Although it seems that the set of terminal distributions obtained should still be the same, our proof requires a further condition of some sort. Theorem 5.1 gives one such condition and another is explained in the discussion which follows its proof.

*Research supported by National Science Foundation Grant DMS-9703285.