THE ESTIMATION OF $p \geq q$ FROM TWO INDEPENDENT BINOMIAL SAMPLES $b(n,p)$ AND $b(m,q)$

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ABSTRACT. The estimation of the probability of success p from independent binomial samples $b(p,n)$ and $b(q,m)$ where $p \geq q$ is considered using a likelihood approach.

1. INTRODUCTION

Consider the model $\mathcal{B}$ of two independent binomial samples $x \sim b(n,p)$, $y \sim b(m,q)$ under the restriction $p \geq q$. The problem is to make quantitative statements about p, based on the information contained in both samples.

The focus of previous approaches to this problem has been on point estimates, for instance the maximum likelihood estimate (mle), and their optimal properties, such as mean squared error (mse), bias, variance, admissible and minimax estimators, the use of certain loss functions, etc. See, for example, Robertson and Waltman (1968), Sackrowitz (1970), Johnson (1971), and Hengartner (1999). One conclusion of these approaches has been that in order to have higher precision for estimating p, depending on the value of q, y should be discarded.

Here the consequences of basing inferences about p on the whole observed likelihood function based on both samples are examined. The use of the whole likelihood function, and not just its mle, has been increasingly used in problems of estimation since Fisher (1956, p. 73), (*e.g.* Barnard, Jenkins and Winston (1963), Edwards (1992), Sprott (2000)). The purpose is to make quantitative statements of uncertainty about unknown parameters based on all of the sample information. "The likelihood supplies a natural order of preference among the possibilities under consideration", Fisher (1956, p. 73). For a single parameter the results can usually be given in the form of graph of the likelihood function supplemented by a set of nested likelihood intervals – see Sprott (2000, Section 2.8) – as in Figure 1.