WHAT IS OPTIMALITY IN SCIENTIFIC INFERENCE?

DAVID A. SPROTT

1. Introduction

The following are only two examples of an increasing tendency to tie scientific inferences to goals or decisions by imposing diverse extraneous optimality criteria:

(1) *"Thirteen methods for computing binomial confidence intervals are compared based on their coverage properties, widths, and errors relative to exact limits." Statistics in Medicine 12 (1993), 809-823.*

(2) *"Six different statistical methods for comparing limiting dilution assays were evaluated, using both real data and a power analysis of simulated data." In Vitro Cellular & Development Biology 25 (1989), 69-75.* These methods depended on specific statistical goals e.g. minimizing type 1 errors vs. maximizing the ability to discriminate between treatments.

It is reasonable to have six different methods for designing an experiment depending on specific scientific goals. But once the experiment has been performed, yielding an observed sample S_o, it seems contradictory to produce six different quantitative statements of uncertainty about values of θ. The resulting complexity is noteworthy.

It is assumed that science is the study of repeatable phenomena. Its purpose is to predict nature. This requires reproducible experiments. This leads to statistical models of experiments in the form of probability functions $P(S_o; \theta)$ of the observations S_o, usually in terms of unknown parameters θ. The primary problem of inference to which this leads is that of inferential estimation. This consists of specific quantitative statements of the plausibility or support by the observed data S_o of the various possible values of θ. A typical example is $\theta = \bar{y} \pm st_{(n-1)}$ appropriate for normal observations.

KEY WORDS: Bayesian intervals; bias; likelihood intervals; likelihood-confidence intervals; maximum likelihood; normal likelihoods; shortest intervals.