SECOND ORDER ASYMPTOTICS FOR M-ESTIMATORS UNDER NON-STANDARD CONDITIONS

NICOLAS W. HENGARTNER AND MARTEN H. WEGKAMP

ABSTRACT. This paper establishes, under non-standard conditions, an explicit stochastic approximation of studentized M-estimators $\widehat{\theta}_n$, implicitly defined as solutions to $\sum_{j=1}^n \psi(X_j, \widehat{\theta}) = o(n^{-1})$, by a U-statistic U_n that is probably concentrated about $\hat{\theta}_n$ in the sense that $\mathbb{P}[|\hat{\theta}_n - U_n| > (n \log n)^{-1}] = o(n^{-1/2})$. The expansion and concentration hold under weaker smoothness conditions on ψ than those assumed by Lahiri (1994). This approximation is key in rigorously establishing a second order expansion for the sampling distribution of the studentized estimator. Under stronger smoothness assumptions on ψ, a similar expansion relates the bootstrap approximation to the true distribution of the studentized M-estimator.

1. INTRODUCTION

Let $X_1, \ldots, X_n$ be independent and identically distributed random variables from a common probability measure P, of which we want to estimate the population parameter $\theta = \theta_P$ implicitly defined as the unique solution of

$$\int \psi(x, \theta_P) dP(x) = 0, \tag{1.1}$$

for some measurable function $\psi(x, \theta)$. For ease of exposition, we shall restrict our attention to univariate parameters. Generalization to the multidimensional parameter case is straightforward, and therefore will not be pursued here.

Denote by P_n the empirical probability measure based on the sample $X_1, \ldots, X_n$ which assigns mass $P_n(A) = n^{-1}\#(X_j \in A)$ to each set A. This paper studies M-estimators $\hat{\theta}$ that solve the empirical counterpart of (1.1) within $o(n^{-1})$, e.g.,

$$\int \psi(x, \hat{\theta})\, dP_n(x) = o(n^{-1}). \tag{1.2}$$

Key words and key phrases: Bootstrap, Edgeworth expansion, M-estimation, Second order asymptotics, Stochastic approximation.

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