

# CHAPTER 11

## Type III Problems: Global Slice

In Type III problems there is, as in Type I problems, a group  $H$  such that  $K = GH$  is a group transitive over  $\mathcal{X}$  but not all items of Assumption 8.11 are satisfied. All assumptions except 8.11(ii) are rather mild and can be expected to hold in applications. On the other hand, Assumption 8.11(ii) turns out to be fairly strong and can easily fail. Inspection of the examples in Chapter 9 reveals that in Section 9.2, Case 1, 8.11(ii) holds because  $G$  and  $H$  commute (and therefore  $G_0$  and  $H$  commute), whereas in all other examples the validity of 8.11(ii) is a consequence of  $G_0 = \{e\}$ . If neither  $G_0$  and  $H$  commute nor  $G_0 = \{e\}$ , then 8.11(ii) is likely to fail. In that case  $\mathcal{Z} = Hx_0$  need no longer be a cross section and it seems as though the additional structure provided by the group  $H$  is useless. However, it turns out that  $H$  and  $\mathcal{Z}$  can still be useful provided a different kind of group structure exists. Consider:

11.1. ASSUMPTION. *Let Assumption 8.11 be satisfied except that 8.11(ii) is changed to*

$$(ii)' \quad gH_0g^{-1} = H_0 \text{ for every } g \in G,$$

*and in addition assume*

$$(iii) \quad gHg^{-1} = H \text{ for every } g \in G.$$

Thus, Assumption 11.1 is Assumption 8.11 with  $G$  and  $H$  interchanged (note that in 8.11 all assumptions except (ii) are symmetric in  $G$  and  $H$ ), and in addition  $H$  is assumed to be normal in  $K$ .