CHAPTER 10

Application to Type II Problems: No Special Group Structure, But Global Cross Section Exists

10.1. Characteristic roots of a positive definite matrix. Let $S \in PD(p)$ be a random matrix with distinct characteristic roots $\lambda_1 > \cdots > \lambda_p > 0$ and distribution $p(S)(dS)$ as in Examples 8.1 and 8.7. The results of Example 8.7 can be copied by changing n to p, with the results

$$(dS) = 2^{-p} \prod_{i<j} (\lambda_i - \lambda_j) \mu_{\mathcal{Y}}(dy)(d\Lambda) \quad \text{at} \quad S = \Lambda, \tag{10.1.1}$$

$$P(d\Lambda) = 2^{-p} \prod_{i<j} (\lambda_i - \lambda_j)(d\Lambda) \int p(\Gamma \Lambda \Gamma') \mu_{O(p)}(d\Gamma). \tag{10.1.2}$$

Equation (10.1.2) can also be found in Muirhead (1982), Theorem 3.2. 17, (but note that Muirhead's Haar measure on $O(p)$ is normalized). If in (10.1.2) $p(S)$ depends on S only through Λ, then formula (2) of Theorem 13.2.1 in Anderson (1984) is reproduced. In particular, if $S \sim W(n, I_p)$ (take formula (9.2.8) with $\Sigma = I_p$), then the result is

$$\begin{aligned} P(d\Lambda) = {} & (2\pi)^{-\frac{1}{2}pn} 2^{-2p} c_n c_{n-p}^{-1} c_p \\ & \cdot \prod_{i<j} (\lambda_i - \lambda_j) |\Lambda|^{\frac{1}{2}(n-p-1)} e^{-\frac{1}{2} \operatorname{tr} \Lambda} (d\Lambda) \end{aligned} \tag{10.1.3}$$