CHAPTER 8

Factorization of Measures on Locally Compact Spaces Induced by the Action of a Group, with Help of a Global Cross Section: Theory

From this chapter on, the notation will revert to that of Chapter 1. That is, unlike the notation in Chapters 2–7, spaces will be denoted by script symbols such as $\mathcal{X}$, $\mathcal{Y}$, etc., and random variables by capital symbols such as X, Y, etc. Free use will be made of concepts and definitions in Chapter 2–7 without always giving a reference. All spaces will be locally compact (l.c.), and a measure on such a space will always be understood to be in the Bourbaki sense (Chapter 6); in particular, a measure is regular and finite on compacta.

Suppose that a statistical problem leads to a random variable X with values in a l.c. space $\mathcal{X}$ and having a distribution P that is a member of some family of distributions. We shall assume that this family is absolutely continuous with respect to a measure λ on $\mathcal{X}$, and write $P(dx) = p(x)\lambda(dx)$. Suppose the statistical problem is invariant under the left action of a l.c. group G (what that means exactly depends on the type of problem and is irrelevant for the present discussion) and suppose we would like to obtain a factorization of λ induced by G as described in Chapter 1. The main result of such a factorization is that it leads to the distribution of a maximal invariant.