CHAPTER 7

Invariant and Relatively Invariant Measures on Locally Compact Groups and Spaces

The various definitions and properties of measures are handled in this chapter in terms of continuous functions with compact support, in the spirit of the Bourbaki integration theory. The results hold then also for the integrable functions. This will be understood tacitly and will generally not be mentioned further.

In Sections 7.1 and 7.2 frequent reference will be made to Nachbin (1976). For short this will often be abbreviated "N."

7.1. Haar measure on locally compact groups. Let G be a l.c. group and $\mathcal{K}(G)$ the family of real valued continuous functions on G with compact support. Since G is l.c., the theory of Chapter 6 applies and any measure on G will be understood to be in the sense of Section 6.3. Consider the left and the right action of G on itself and recall the definition of the left g-translate gf and the right translate fg of a function f on G by an element $g \in G$ (Section 2.1).

7.1.1. DEFINITION. *A measure μ on G is called* **left invariant,** *or a* **left Haar measure,** *if*

$$\mu(gf) = \mu(f), \quad g \in G, \ f \in \mathcal{K}(G). \tag{7.1.1}$$