CHAPTER 6

Integration on Locally Compact Spaces According to Bourbaki

For convenience of reference the following two references will be abbreviated: Bourbaki (1965) by "B," and Taylor (1965, 1985) by "T." The reader is reminded of the symbol $\mathcal{K}(X)$ introduced in Section 2.2 for the family of real valued continuous functions with compact support on the locally compact (l.c.) space X.

6.1. The Daniell method. There are basically two very different theories of measure and integration on a given space X. In the first one, which will be called "classical" here, the starting point is a family of subsets of X, called measurable, on which a measure is defined as a set function with certain properties. This theory is documented very well in Halmos (1950). The second approach is due to Daniell (1917–18, 1919–20) and consists of first defining the integral as a linear functional, with a certain monotonicity and continuity property, on a family of "nice" functions; then extending the integral to a wider family of functions called "integrable," and finally defining measurable functions and sets. Thus, in the Daniell approach integrable functions and their integrals come first, measurable sets come last, in contrast to the classical approach. An advantage of the Daniell approach is that certain properties of the integral already follow from