CHAPTER 3

Differentiable Manifolds, Tangent Spaces, and Vector Fields

This chapter touches mostly on the topics that are relevant to the later applications in this monograph. For other important topics in differential geometry, for instance fibre bundles, connections, Riemann metric, curvature, etc., the reader is referred to the literature in this field; see, e.g., Bishop and Crittenden (1964), or Greup, Halperin, and Vanstone (1972). For applications of differential geometry to statistical parameter spaces see Amari, Barndorff-Nielsen, Kass, Lauritzen, and Rao (1987).

3.1. Manifolds. The spaces and groups encountered in this monograph have more structure than merely being topological: they are manifolds. Loosely speaking, a manifold is a space that is locally Euclidean at each point. A trivial example is a Euclidean space itself. More interesting examples are curved subsets of Euclidean spaces. For instance, the parabola $x_2 = x_1^2$ is a one-dimensional manifold embedded in R^2, and the sphere $x_1^2 + x_2^2 + x_3^3 = 1$ is a two-dimensional manifold embedded in R^3. But the subset $\{(x_1, x_2) : x_1 x_2 = 0\}$ of R^2 is not a manifold because the point $(0, 0)$ does not have a Euclidean neighborhood.