CHAPTER 2

Spaces, Functions, and Groups Acting on Spaces

This chapter is divided into three sections. In Section 2.1 notions are introduced that do not involve the concept of continuity. Topology is introduced in Section 2.2 and, among other things, proper mappings are defined. The most interesting questions deal with the interaction between algebra and topology, and these are treated in Section 2.3. For simplicity of notation in this and several subsequent chapters, we shall denote spaces by symbols X, Y, etc., instead of $\mathfrak{X}$, $\mathfrak{Y}$, etc. From Chapter 8 on we shall revert to the latter notation and reserve X, Y, etc. for random variables, as in Chapter 1.

2.1. Spaces, functions, groups, and group action. Let X and Y be two arbitrary spaces and f a function $X \to Y$. Instead of "function" the names **mapping** or **map** are also used. The **range** of f is range $f = \{f(x) : x \in X\}$. If range $f = Y$, f is said to be **onto**, or **surjective**. If $f(x_1) = f(x_2)$ implies $x_1 = x_2$ for all $x_1, x_2 \in X$, then f is called **one-to-one** (or 1-1), or **injective**. In that case f^{-1} is defined as a function on range f onto X. If f is both 1-1 and onto, it is also called **bijective**. In that case there is a 1-1 correspondence between X and Y, and f^{-1} is defined on all of Y. For arbitrary $f : X \to Y$, f^{-1} is always defined as a set function: for $B \subset Y$,