

# CHAPTER 1

## Introduction

This monograph deals with problems concerning distributions in statistical models in which there is a group of invariance transformations. The methods to be presented make use of mathematical tools that involve interplay between groups and integration. The purpose of this monograph is not only to demonstrate by examples the statistical usefulness of the methods, but also to present a systematic account of the mathematical background.

One of the most important problems in a statistical model with an invariance group is to obtain the distribution of a maximal invariant (precise definitions of this and other notions will be made in subsequent chapters). However, other statistics that are not invariant but equivariant may be of interest too (for instance, maximum likelihood estimators). Often these distributional problems can be handled without group methods on an ad hoc basis, but the full use of a known invariant measure on the group provides a more systematic treatment that usually also is easier to carry out. Invariant measures are guaranteed for locally compact groups, and it is to those groups that this monograph will be confined. That includes all such “nice” groups as translations and matrix groups, but not large groups such as all continuous and strictly monotonic transformations of the real line. Thus, the statistical problems to be considered are typically of the parametric rather than nonparametric type.

As a justification for the use of group methods in a simple example consider the derivation of the  $\chi^2$ -distribution, or equivalently,