

LIMIT THEOREMS FOR RANDOM CENTRAL ORDER STATISTICS

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Let  $X_{n,1} \leq \dots \leq X_{n,n}$  be the order statistics of a random sample  $X_1, \dots, X_n$  from a distribution function  $F$ . If  $\{v_n\}$  is a sequence of integer-valued random variables such that  $1 \leq v_n \leq n$  and  $v_n/n \xrightarrow{P} p$ , for some  $p \in (0,1)$ , the sequence  $\{X_{n,v_n}\}$  is referred to as a sequence of random central order statistics (of limiting rank  $p$ ) corresponding to the random central rank sequence  $\{v_n\}$ . In this paper, we establish the weak as well as strong consistency of  $X_{n,v_n}$  in estimating the  $p$ -th quantile of  $F$ . We derive several central limit theorems for  $X_{n,v_n}$  for regular as well as non-regular cases, and for each case, we provide remainder term estimates of the Berry-Esséen type.

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