

REMARKS ON THE ESTIMATION OF COEFFICIENTS OF A REGRESSION  
IN THE PRESENCE OF UNKNOWN EXPLANATORY VARIABLES

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In the linear regression model  $Y = \beta_1 X_1 + \beta_2 X_2 + u$ , the coefficients  $\beta_1$  and  $\beta_2$  may be estimated by least squares. If the explanatory variable  $X_2$  is not observed, the regression of  $Y$  on  $X_1$  will give an estimate of  $\beta_1$  whose bias will depend on the correlation between  $X_1$  and  $X_2$ . However qualitative knowledge about  $X_2$  can be exploited. We treat the case where the known and unknown explanatory variables and the coefficients are nonnegative and where it is known that for some, but not which, data points, the unknown explanatory variables are relatively small.

1. Introduction.

A source of difficulty in estimating the effect of one variable on another, especially in observational studies, is that the explanatory model may omit a causal variable. Under some circumstances, this difficulty may be serious. If the omitted variable is unimportant, i.e. it has a relatively small effect, it may be safe to ignore it. If it is uncorrelated with the other explanatory variables, it may also be ignored in linear regression models. If it is correlated with the explanatory variables, and one desires only to use these for prediction, one may proceed without it, as long as that correlation is

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