

DISTRIBUTION OPTIMALITY AND SECOND-ORDER EFFICIENCY*
OF TEST PROCEDURES

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It has been shown, under certain conditions, by Bahadur, Chandra, and Lambert (1982) that in the non-null case the best possible asymptotic distribution for the level attained by a test statistic is a certain lognormal distribution, and that the level of the likelihood ratio statistic has this optimal asymptotic distribution. We describe a technical generalization of this theory; in the present generalization the best possible asymptotic distribution of the standardized log-level is that of the maximum of a family of normally distributed variables. It is pointed out that these considerations yield a corresponding generalization concerning the asymptotic expansion of the log-size of the best critical region when the power against a given alternative is a specified constant.

1. Introduction.

In the following sections S is a sample space of points s , and \mathcal{A} is a σ -field of subsets of S . Θ is a parameter space of points θ and, for each

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