

MAXIMUM LIKELIHOOD RECURSION AND STOCHASTIC

APPROXIMATION IN SEQUENTIAL DESIGNS

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An approach to sequential design for estimating the root of a nonlinear equation is described. It sets the next design point at the current estimate of the parameter via a parametric model and maximum likelihood (or other efficient) estimation. For normal, binomial and Poisson errors and their respective canonical link functions, it is close to the Robbins-Monro stochastic approximation and thus enjoys the latter's robustness against the misspecification of the link function. Some new variations of the Robbins-Monro scheme are obtained as a consequence.

1. Introduction.

We are interested in efficient sequential designs for estimating the root of an unknown nonlinear equation, where the distribution of the responses is quite general (continuous or discrete). The proposed approach is based on design updating with the maximum likelihood estimate via a parametric model. It is dubbed the maximum likelihood (ML) recursion approach. In several important situations it is shown to be closely related to the stochastic approximation approach of Robbins and Monro (1951).

The problem can be described as follows. The response y is related to an underlying "design" variable x . Denote the mean of y at x by $M(x)$, which is

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