

## STOCHASTIC APPROXIMATION AND ADAPTIVE CONTROL

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The pioneering work of Robbins and Monro in stochastic approximation initiated a rich and important field of recursive procedures in estimation and control for stochastic models. Herein the control aspects of Robbins-Monro schemes are discussed. In this connection, we also review some recent results on (i) adaptive stochastic approximation, (ii) its extensions to general multivariate stochastic regression models, and (iii) the application of these extensions to the classical problem of adaptive control of linear dynamic systems and time series models.

### 1. Stochastic approximation.

In 1951, Robbins and Monro introduced the subject of stochastic approximation in their seminal paper on the problem of finding the root of a regression function by successive approximations. Consider the regression model

$$(1.1) \quad y_i = M(x_i) + \epsilon_i \quad (i=1,2,\dots),$$

where  $y_i$  denotes the response at the design level  $x_i$ ,  $M$  is an unknown regression function, and  $\epsilon_i$  represents unobservable noise (error). In the deterministic case (where  $\epsilon_i = 0$  for all  $i$ ), Newton's method for finding the root  $\lambda$  of a smooth function  $M$  is a sequential scheme defined by the recursion

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