CONSTRAINED STOCHASTIC APPROXIMATION

VIA THE THEORY OF LARGE DEVIATIONS

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Let G be a compact convex subset of R^r, $\{\xi_i\}$ a bounded random sequence, and $\Pi_G(x)$ the projection of x onto G. We obtain asymptotic properties of the projected stochastic approximation algorithm $X^\varepsilon_{n+1} =$

$$\Pi_G(X^\varepsilon_n + \varepsilon b(X^\varepsilon_n,\xi_n)) \text{ (or } X_{n+1} = \Pi_G(X_n + a_n b(X_n,\xi_n)), a_n \to 0),$$

via the theory of large deviations. The action functionals and their properties are obtained, as is the mean exit time from a neighborhood of a stable point of the 'mean' algorithm. The usual methods for obtaining the 'asymptotic normality' of suitably centered and normed sequences in stochastic approximation do not work here - and, in fact, this (asymptotic normality) property would not usually hold. The large deviations approach provides a useful alternative. Even for the unconstrained case,for many applications the large deviations estimates seem to be more useful than those based on the 'local linearization' which leads to the asymptotic normality.

1. Introduction.

Let $G = \{x:q_i(x) \leq 0, i \leq k\}$ be a compact convex subset of R^r which is the closure of its interior, where the $q_i(\cdot)$ are continuously differentiable.

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