STOCHASTIC APPROXIMATION FOR FUNCTIONALS

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Suppose $\underline{F}$ is a class of distributions containing the discrete distributions and the distribution F_x for each real x. Suppose ϕ is a real valued functional on $\underline{F}$ and define $\theta(x) = \phi(F_x)$ so that $\theta(.)$ is a parameter of the family $\{F_x\}$. Fix α. A stochastic approximation procedure for finding the x for which $\theta(x) = \alpha$ is presented. When $\phi(F)$ is the mean of F, a form of this procedure is just the Robbins-Monro process. When $\phi(F)$ is the p-th quantile of F, a form of this procedure is just the quantile process introduced by the authors in an earlier paper. Some convergence theorems, examples, and generalizations are presented.

1. Introduction.

Suppose that for each real x (or for each x in some interval) there is a distribution F_x from which we can sample at will. Suppose $\underline{F}$ is a collection of distribution functions containing all empirical distribution functions (i.e., all distribution functions of the form $F(t) = \frac{1}{n}\sum_{k=1}^{n} I_{[a_k,\infty)}(t)$) and all of the distribution functions F_x. Let ϕ be a real valued functional on $\underline{F}$ and define $\theta(x) = \phi(F_x)$ so that $\theta(.)$ is some parameter of the family $\{F_x\}$. Our objective

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