THE FINITE STATE COMPOUND DECISION PROBLEM, EQUIVARIANCE AND RESTRICTED RISK COMPONENTS*

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The envelope results of Hannan and Huang (1972a) are generalized to arbitrary bounded risk components with simplified proofs. For equivariant "delete bootstrap" procedures, the excess compound risk over the simple envelope is bounded in terms of the L_1 error of estimation thus establishing a large class of asymptotic solutions to the compound decision problem with restricted risk component. This class includes the compound procedures which are Bayes versus certain symmetric priors (cf. Gilliland, Hannan and Huang, 1976). Asymptotic solutions and Bayes procedures in the empirical Bayes problem follow from those of the compound decision problem.

1. A finite state, restricted compound decision problem.

Consider a decision problem with states $P \in \mathcal{P} = \{F_0, F_1, \ldots, F_m\}$, where the F_i are distinct probability measures on $(\mathcal{X}, \mathcal{B})$. Let the risk set S of the decision problem be a bounded subset of $[0,\infty)^{m+1}$ and let $s = (s_0, s_1, \ldots, s_m)$ denote a generic point of S.

Consider a compound decision problem involving N independent repetitions of the above component structure. For $\underline{x} \in \mathcal{X}^N$ let $\check{\underline{x}}_\alpha$ denote $\underline{x}$ with the α-th component deleted and for $\underline{P} = \overset{N}{\underset{\alpha=1}{\times}} P_\alpha$ with $P_\alpha \in \mathcal{P}$, let $\check{\underline{P}}_\alpha$ denote $\underline{P}$ with the α-th factor deleted, $\alpha = 1,2,\ldots,N$. Consider the class $\underline{S}$ of the compound rules $\underline{s} = (\underline{s}_1, \underline{s}_2, \ldots, \underline{s}_N)$ where for each α, $\underline{s}_\alpha = (\underline{s}_{\alpha 0}, \underline{s}_{\alpha 1}, \ldots, \underline{s}_{\alpha m})$ denotes a

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