CONVERGENCE RATES FOR ITERATIVE SOLUTIONS TO OPTIMAL STOPPING PROBLEMS

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We transform a class of optimal stopping problems to problems of finding fixed points of certain transformations, and show that the classical iterative solutions to the latter yield approximations to the value of the corresponding optimal stopping problem. We give upper and lower bounds on the value as functions of the number of iterations, and numerically treat two examples: the well known S_n/n problem, and the coupon collector problem, which is solved exactly and numerically compared to the iterative approximations.

1. Iterative solutions to fixed point problems.

If Q is an operator which sends a space B into itself, a point x is called a fixed point if Qx = x. Many problems, both theoretical and applied, are those of finding the fixed points of certain transformations. For example, the extinction probability and the characteristic functions of some limiting distributions in branching theory, the limiting renewal function in renewal theory, certain optimizing strategies, and the equilibrium states of certain evolving phenomena are fixed points.

The iterative method of finding fixed points consists of taking an initial value x_0, arrived at in some manner, and determining sequentially x_n by setting $x_{n+1} = Qx_n$, n = 0,1,... . If, in some fashion, x_n "converges" to a limit x, it is generally possible to show that x is a fixed point for Q. The main questions here are: 1) Does a fixed point exist, and if so is it unique?

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