ON THE PASSAGE OF A RANDOM WALK FROM GENERALIZED BALLS*

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We derive a strong approximation for the first passage time of zero-mean random walks from generalized balls in Euclidean spaces by the corresponding first passage time of an appropriate vector valued Wiener process as the radius of the ball goes to infinity. As consequences we derive a weak invariance principle and some strong laws for the passage time of the walk. Multidimensional extensions of some limit theorems of Robbins and Siegmund on boundary crossing probabilities for sample sums are also formulated, including their last-time result.

1. Introduction.

Let $d \geq 1$ be a fixed integer and consider a sequence $X, X_1, X_2, \ldots$ of independent and identically distributed random vectors with values in $\mathbf{R}^d$. Introduce the corresponding partial sum process, or continuous-time random walk

$$S(t) = \sum_{i=1}^{[t]} X_i, \quad t \geq 0,$$

where $[.]$ denotes integer part and $S(t) = 0$ for $0 \leq t < 1$, and let $h: \mathbf{R}^d \to \mathbf{R}$ be a

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