# 6. APPROXIMATE CONFIDENCE INTERVALS IN MULTIPARAMETER PROBLEMS

## 6.1. INTRODUCTION

As has been noted in the previous chapters, a limiting aspect of the saddlepoint approximation has been the computational complexity in higher dimensions. In a problem with $p$ parameters, we are often interested in approximating the marginal density of the estimate of one of the parameters or the density of some test statistics. We could in principle use the approximation to obtain the density on a grid in p-dimensions and then integrate over appropriate regions of p-dimensional space. However, at each grid point, we have to solve a non-linear system of $p$ equations to obtain $\alpha$. If $p$ is three or more this is not really a feasible method numerically.

In this chapter, we introduce a technique developed by Tingley and Field (1990) designed to overcome this problem. In fact the approach to be presented will be based on a nonparametric bootstrap and will allow us to obtain correct one or two-sided second order confidence intervals without specifying an underlying density. One of the tools that is essential is the tail area approximation due to Lugannani and Rice (1980). This approximation eliminates the integration of the approximate density in calculating tail areas.

The next section discusses the tail area approximation while the third section demonstrates how to compute confidence intervals which are both robust and nonparametric.

## 6.2. TAIL AREA APPROXIMATION

Up to this point, we have developed approximations for densities of estimates. However in many situations, there is more interest in approximating the cumulative distribution. In particular, for confidence intervals and testing procedures, it is the tail area of the distribution which is of interest. In this section we will develop a tail area approximation for the case of the univariate mean. This approximation will then be used in the next section to construct confidence intervals in the multiparameter situation. The tail area approximation is based on uniform asymptotic expansions and was developed for tail areas by Lugannani and Rice (1980). Both Daniels (1987) and Tingley (1987) have placed the result in the context of small sample or saddlepoint approximations. Our development is similar to that of Daniels and Tingley but with some notational changes.

Consider the situation where we have $n$ independent, identically distributed random variables, $X_1, X_2, \cdots, X_n$ and we want to approximate $P(\bar{X} \geq x_0)$ for some point $x_0$. Based on our previous approximation for the density of $\bar{X}$ we could approximate the upper tail area by $\int_{x_0}^{\infty} kc^{-n}(x)/\sigma(x)dx$ where $k$ is the normalizing constant. In order to evaluate this integral, we have to evaluate $c(x)$ and $\sigma(x)$ (and hence $\alpha(x)$) over a grid of points from $x_0$ to $\infty$. As noted by Daniels (1987), the integration can be made simpler by a change of variables. The upper tail area can be written as $\int_{x_0}^{\infty} kc^{-n}(\alpha(x))/\sigma(\alpha(x))dx$. The monotone transformation $y = \alpha(x)$, gives the integral $\int_{\alpha(x_0)}^{\infty} kc^{-n}(y)\sigma(y)dy$ which avoids the necessity of computing the saddlepoint $\alpha(x)$ for every ordinate. Related work by Robinson (1982) gives a tail area approximation based on the Laplace approximation to the integral above.