5. RELATED TECHNIQUES

5.1 INTRODUCTION

In this chapter, a number of related techniques will be presented emphasizing their relationship to small sample asymptotics. Section 2 looks at an approach developed by Frank Hampel which has a number of desirable properties.

Next the relationship of small sample asymptotics to saddlepoint and large deviations is presented. We then turn to the work of Durbin and Barndorff-Nielsen and attempt to relate their work in the case of sufficiency and/or exponential families to the techniques of small sample asymptotics. To conclude the chapter, computations are done in the case of logistic regression to contrast the various approaches.

5.2. HAMPEL'S TECHNIQUE

In the paper, Hampel (1973), many of the motivating ideas for small sample asymptotics are laid down. Both authors were introduced to the topic via the paper and it is important to acknowledge its influence. Although the results turn out to be closely related to saddlepoint results, they were developed independently of the saddlepoint work of Daniels (1954). The approach proposed by Hampel is very interesting and probably has yet to be fully exploited. Our purpose here is to present the ideas and suggest some possible future directions. The initial development follows Hampel (1973) very closely, especially p. 111, 112.

Hampel's approach differs in several ways from typical classical approaches. The first is that the density of the estimate, rather than a standardized version of it, is approximated. A second feature is that we use low-order expansion in each point separately and then integrate the results rather than use a high-order expansion around a single point. It is this feature which really distinguishes small sample asymptotics (and saddlepoint techniques) from classical asymptotic expansions. The local accuracy from the first one or two terms is effectively transferred to a selected grid of points yielding the same accuracy globally. It is the availability of cheap computing which makes feasible this use of local techniques. A fairly simple approximation requiring non-trivial computation is carried out at a number of grid points. This is of course exactly the type of problem which is ideally suited to computer computations.

The third difference concerns the question of what to expand. Hampel argues effectively that the most natural and simple quantity to study is the derivative of the logarithm of the density, namely f_n'/f_n. There are at least four reasons why this seems reasonable.

(i) The form of the expansion of f_n'/f_n is such that the first term is proportional to n and the first two terms are linear in n. This contrasts with more complicated relationships coming from f_n or the cumulative.

(ii) Since our expansions are local in nature, it makes sense to focus on a feature of a distribution which is not affected by shifts or addition or deletion of mass elsewhere. Neither f_n or the cumulative satisfy these properties. f_n'/f_n is the first and simplest quantity with these local properties.

(iii) We can view the normal distribution as playing a very special and basic role in probability, in many ways analogous to the role of the straight line in geometry. For the normal, it is f'/f which has a particularly simple form, namely a linear function of x. By expanding f_n'/f_n locally, we are, in a sense, linearizing a function locally.