VI. AFTERTHOUGHTS

1. Two topics that were left out.

THE $\mathcal{L}^2$ SPACE OF A GAUSSIAN PROCESS: Looking back over what I have written, it seems to me that these notes contain almost everything a beginning researcher needs to know about the mathematical basis of Gaussian processes. Somehow, although my original intention was only to cover problems of continuity and extrema, there is a lot of other material that has found its way into the notes. There is one gaping omission, however, and this relates to the $\mathcal{L}^2$ space associated with a Gaussian process, and its Hilbert space structure.

The reason that this material does not appear here is that it has nothing to do with continuity, boundedness, and suprema distributions, and these have been our main concern. Nevertheless, a word of advice to the teacher or the reader: If, after going through these notes you have some time and energy left, read the first seven chapters of Major's (1981) monograph on multiple Wiener integrals, and follow it up with the elegant and powerful central limit theorem of Dynkin and Mandelbaum (1983). Then you will be ready to start a serious reading of the Gaussian literature in all its breadth and beauty.

ON MARKOV PROCESSES AND THE ISOMORPHISM THEOREM: Markov processes have appeared in these notes only in a very peripheral fashion. This is unfortunate, because it tends to reinforce the "well known fact" that Gaussian and Markovian processes have little to do with one another, and this WKF is taking somewhat of a beating at the moment. To explain why, I want to describe for you a simple version of a theorem due to Dynkin (1983). Then I will explain to you why this theorem is interesting.

Let $\{X_t, t \geq 0\}$ be an $\Re^d$-valued, symmetric, Markov process with stationary transition density $p_t(x,y) = p_t(y,x)$. (We really need certain technical side conditions, but shall leave them out here.) Let ξ be an exponential random variable with mean one, independent of X, which we treat as a death time for X, and let Δ be the "cemetary" state for X so that the "killed version" of X is given by the process

$$\hat{X}_t = \begin{cases} X_t, & t < \xi, \\ \Delta, & t \geq \xi. \end{cases}$$

The killed process $\hat{X}_t$ is still a Markov process, with transition density