I. INTRODUCTION

1. The Basic Ideas.

It is my aim in these notes to treat two basic and closely related problems in the theory of Gaussian processes: the question of sample path continuity and the distribution of the supremum of a Gaussian process over a fixed set in its parameter space.

The basic approach will follow the modern attitude that the precise geometrical structure of the parameter space of a Gaussian process is of relatively little importance in determining sample path properties. Of more importance is establishing the "size" of the parameter space when measured in terms of a metric based on the covariance function of the process. There are various ways to measure this size, the best known, and easiest to handle, being via the notion of *metric entropy*. The most recent, and most powerful tool however is the notion of *mesure majorante*, or *majorising measure*, introduced by Preston (1972), (albeit not under this name), developed by Fernique in a series of papers starting with Fernique (1974), and recently shown by Talagrand (1987) to be the only tool currently available that provides necessary, as well as sufficient, conditions for the a.s. continuity of a general Gaussian process.

In between regular metric entropy and majorising measures lie some other notions, retaining the ease of application of metric entropy while giving stronger results. We shall consider two of these as well, once in studying continuity, and once while looking at suprema distributions.

Before we can start anything, however, we need to settle on terminology and notation.

$(\Omega, \mathcal{F}, P)$ will be a complete probability space that will remain fixed throughout the notes. A Gaussian random variable X with mean $\mu \in \Re$ and variance $\sigma^2 \in \Re_+$ is a real valued random variable such that for each $\lambda \in \Re$

$$Ee^{i\lambda X} = e^{i\mu\lambda - \frac{1}{2}\sigma^2\lambda^2},$$

or, equivalently, the law of X has density $\sigma^{-1}\phi((x-\mu)/\sigma)$, where

$$\phi(x) := (2\pi)^{-\frac{1}{2}} \exp(-x^2/2).$$

If $\mu = 0$ we call X *centered*, and if we also have $\sigma = 1$, then X is called *standard normal.* Since the transition from centered to non-centered Gaussians is via the easy addition of a constant we shall treat, almost exclusively, only centered Gaussian variables.

A (centered) Gaussian process is a family $\{X_t\}_{t\in T}$ of random variables, indexed by a parameter set T, such that each linear combination $\sum \alpha_t X_t$ is (centered) Gaussian. Whereas the use of the letters t and T seem to