

Chapter 5. Examples of Data on Permutations and Homogeneous Spaces

To fix ideas, as well as to make contact with reality, it is useful to have a collection of real data sets on hand.

A. PERMUTATION DATA.

- (1) Large sets of rankings are sometimes generated in psychophysical experiments (rank these sounds for loudness), taste testing experiments (rank these 5 types of coffee ice cream), or surveys. To give an example, in 1972, the National Opinion Research Center included the following question in one of their surveys: Where do you want to live? Rank the following 3 options: in a big city; near a big city (≤ 50 miles); far from a big city (> 50 miles). The data from 1439 respondents was

city	suburbs	country	#
1	2	3	242
1	3	2	28
2	1	3	170
3	1	2	628
2	3	1	12
3	2	1	359

Let us briefly discuss this data. The modal rank is $\frac{1}{3} \frac{2}{1} \frac{3}{2}$ — people prefer the suburbs, then country, then city. This is born out by simple averages: 270 people ranked city first, 798 ranked suburb first, 371 ranked country first.

The 2 small counts lead to an interesting interpretation. Both violate the unfolding hypothesis of Coombs (1964). To spell this out a bit, suppose people's rankings are chosen in accordance with the ideal distance from the city, different people having different preferences. Thus, one chooses the rank one location and then “unfolds” around it. In this model $(\frac{1}{1} \frac{2}{3} \frac{3}{2})$ is impossible since if one most prefers being in the city, one must prefer being close to the city to being far away. The number of permutations of the set $1, 2, \dots, n$ consistent with unfolding is about 2^{n-1} , so many arrangements are ruled out. Unfolding is a nice idea, but distance to the city might not determine things for someone who works in the suburbs and doesn't want to live where they work. If you ask people to rank order temperature for tea (hot, medium, cold), you don't expect the unfolding restriction to hold, but if you ask people to rank order sugar teaspoons $(0, \frac{1}{2}, 1, \frac{3}{2}, 2)$ you do expect the data to be consistent with unfolding.

Further analysis of the distance to cities data is in Chapter 8. Duncan and Brody (1982) discuss these data in some detail.