CHAPTER 7. TAIL PROBABILITIES

In exponential families the probability under θ of a set generally falls off exponentially fast as the distance of the set from $\xi(\theta)$ increases. This section contains several results of this form. The first of these will be improved later, but it is included here because of its simplicity of statement and proof.

Throughout this chapter let $\{P_\theta\}$ be a steep canonical exponential family. (Most of the results hold with possibly minor modifications for non-minimal families, and many also hold for non-steep families.)

FIXED PARAMETER (Via Chebyshev's Inequality)

7.1 Theorem

Fix $\theta_0 \in N^\circ$. Choose ε so that $\{\theta: ||\theta - \theta_0|| \leq \varepsilon\} \subset N^\circ$. Then there exists a constant $c < \infty$, such that

$$\Pr_{\theta_0} H^+(v, \alpha) < c \exp(-\varepsilon\alpha) \tag{1}$$

for all $v \in R^k$ with $||v|| = 1$ and all $\alpha \in R$.

Proof. Let

$$c = \exp(\sup \{\psi(\theta) - \psi(\theta_0): ||\theta - \theta_0|| = \varepsilon\}) \tag{2}$$

and let $\theta_\varepsilon = \theta_0 + \varepsilon v$. Then