

CHAPTER 4. APPLICATIONS

This chapter describes three different general applications of the theory developed so far. The first part of the chapter contains a proof of the information inequality and a proof based on this inequality of Karlin's theorem on admissibility of linear estimators.

The second part of the chapter describes Stein's unbiased estimator of the risk and proves the minimaxity of the James-Stein estimator as a specific application of this unbiased estimate.

The third part of the chapter describes generalized Bayes estimator and contains two principle theorems describing situations in which all admissible estimators are generalized Bayes -- or at least have a representation similar to that of a generalized Bayes procedure. This part of the chapter deals with two basic situations. The first is estimation of the natural parameter under squared error loss, and the second is estimation of the expectation parameter under squared error loss. The so-called conjugate prior play a natural role in this second situation.

The exercises at the end of the chapter contain a non-systematic selection of some of the specific results derivable from the more general development in the body of the chapter.

INFORMATION INEQUALITY

The information inequality -- also known as the Cramer-Rao inequality -- is an easy consequence of Corollary 2.6.

The version to be proved below applies to vector-valued as well as real-valued statistics. For vector-valued statistics one needs the multi-