Chapter 3. Parametrizations

In regular exponential families maximum likelihood estimation is closely related to the so-called mean value parametrization. This parametrization will be described after some brief preliminaries. The relation to maximum likelihood is pursued in Chapter 5.

3.1 Notation

For $v \in R^k$, $\alpha \in R$ let $H(v, \alpha)$ denote the hyperplane

$$H(v, \alpha) = \{x \in R^k : v \cdot x = \alpha\} \quad .$$

Let $H^+(a, \alpha)$ and $H^-(a, \alpha)$ be the open half spaces

$$H^+(v, \alpha) = \{x \in R^k : v \cdot x > \alpha\}$$

$$H^-(v, \alpha) = \{x \in R^k : v \cdot x < \alpha\} \quad .$$

When (v, α) are clear from the context they will be omitted from the notation Note that the closure of $H^{\pm}$ is written $\overline{H}^{\pm}$ and, of course, satisfies $\overline{H}^{\pm} = H \cup H^{\pm}$.

STEEP FAMILIES

Most exponential families occurring in practice are regular (i.e. N is open). However, for technical reasons which will become clear in Chapter 6, it is very useful to prove the parametrization Theorem 3.6 for steep families as well.