CHAPTER 1. BASIC PROPERTIES

TANDARD EXPONENTIAL FAMILIES

.1 Definitions (Standard Exponential Family): Let ν be a σ-finite measure n the Borel subsets of R^k. Let

1)
$$N = N_\nu = \{\theta: \int e^{\theta\cdot x}\nu(dx) < \infty\} \quad .$$

et

2)
$$\lambda(\theta) = \int e^{\theta\cdot x}\nu(dx)$$

Define $\lambda(\theta) = \infty$ if the integral in (2) is infinite.) Let

$$\psi(\theta) = \log \lambda(\theta) \ ,$$

nd define

3)
$$p_\theta(x) = \exp(\theta\cdot x - \psi(\theta)) \ , \qquad \theta \in N$$

et $\Theta \subseteq N$. The family of probability densities

$$\{p_\theta : \theta \in \Theta\}$$

s called a k-dimensional *standard exponential family* (of probability ensities). The associated distributions

$$P_\theta(A) = \int_A p_\theta(x)\nu(dx) \ , \quad \theta \in \Theta$$

re also referred to as a standard exponential family (of probability istributions).

N is called the *natural parameter space*. ψ has many names. We will call it the *log Laplace transform* (of ν) or the *cumulant generating function*. $\theta \in \Theta$ is sometimes referred to as a *canonical parameter*, and