# LECTURE XIII. AN APPLICATION TO THE THEORY OF RANDOM GRAPHS

Consider a random graph $G(n)$ on $n$ vertices in which each possible edge is present with probability $p$, independently of all others. Let $W_{n,k}$ (also abbreviated $W_n$) be the number of isolated trees of order $k$ in $G(n)$. Conditions are given for $W_n$ to have approximately a Poisson distribution. This lecture is based on a paper of Barbour (1982), who also gave conditions for a normal approximation to be valid.

I shall use essentially the same notation as Barbour. Denoting the set of vertices by $\{1,\ldots,n\}$, I shall think of the random graph $G(n)$ as a random subset of the set of all two-element subsets $\{i,j\}$ of $\{1,\ldots,n\}$. If $\{i,j\} \in G(n)$ I shall say that $\{i,j\}$ is an edge of the random graph $G(n)$, which will be constructed by having the events $\{\{i,j\} \in G(n)\}$ occur independently with common probability $p$. Let $D_n$ be the set of all $k$-tuples $i = (i_1,i_2,\ldots,i_k)$ of natural numbers with $1 \leq i_1 < i_2 <\ldots< i_k \leq n$. For each $i \in D_n$ let $X_i = 1$ if there is in $G(n)$ an isolated tree spanning the vertices $i_1,\ldots,i_k$, and otherwise let $X_i = 0$. A tree is, by definition, a connected graph containing no cycles, and it is isolated if $G(n)$ has no edge with one vertex in the tree and one not in the tree. Then $W_n$, the number of isolated trees of order $k$ in $G(n)$ is given by

$$W_n = \sum_{i \in D_n} X_i. \tag{1}$$

The expectation $\lambda$ of $W_n$ is given by