LECTURE XI. IMPROVED RESULTS ON THE NUMBER OF LATIN RECTANGLES

In this continuation of the seventh lecture I shall try to present a corrected version of my paper in the 1978 Journal of Combinatorial Theory, Series A. The principal result of that paper is that the number $N_{k,n}$ of $k \times n$ Latin rectangles satisfies

$$(1) \qquad N_{k,n} \sim (n!)^k \prod_{j=1}^{k-1} \left(1 - \frac{j}{n}\right)^n \sim (n!)^k e^{-\frac{k(k-1)}{2} - \frac{k^3}{6n}}$$

as $n \to \infty$, uniformly for $k \leq C\sqrt{n}$, with C an absolute constant. This confirms the conjecture of Erdös and Kaplansky that their formula

$$(2) \qquad N_{k,n} \sim (n!)^k e^{-\frac{k(k-1)}{2}}$$

does not hold beyond $k = o(n^{1/3})$.

Recently Godsil and McKay (1983) have obtained much more precise results by a completely different method. In particular they proved that

$$(3) \qquad N_{k,n} = \left[(n!)^k \prod_{j=1}^{k-1} \left(1 - \frac{j}{n}\right)^n\right]\left(1 - \frac{k}{n}\right)^{-\frac{n}{2}} e^{-\frac{k}{2} + O\left(\frac{k^7}{n^6} + \frac{1}{n}\right)}$$

uniformly for $k = O(n^{1-\delta})$ for any fixed $\delta > 0$, and they suggested the possibility that the error term could be improved to $O(\frac{1}{n} + k(\frac{k}{n})^t)$ for arbitrarily large t. For small k their result is much more precise even than (3). It is not clear whether my method has any hope of yielding results comparable to theirs. Briefly, my approach to improving on the results of Lecture VII is