LECTURE X. ANOTHER ABSTRACT NORMAL APPROXIMATION THEOREM

Here I shall describe an approach that is applicable to the study of sums of m-dependent random variables and to the stationary case with appropriate mixing conditions. In these cases it does not seem possible to construct an exchangeable pair (W,W') with the properties required in Theorem III.1 and in Lemmas I.3, I.4 and III.1, which lead up to that theorem. The method of the present lecture is that which I used in the easier part of my paper on normal approximation in the Proceedings of the Sixth Berkeley Symposium. However, the difficult problem of getting error bounds that are essentially sharp in order of magnitude will not be treated here.

The present treatment is somewhat sketchy. I omit a number of proofs that are completely analogous to proofs in the first three lectures. Also, even the relatively concrete Corollary 2 does not deal with the usual formulation of the problems considered here. Some notion of this can be obtained from my paper mentioned earlier. Finally, I should mention that, in Lecture XIV, I shall reformulate this approach in a way that will clarify its relation to the basic formalism of the first lecture through (I.33).

In order to avoid excessive repetition in stating the hypotheses of the lemmas and theorem of this lecture I shall formulate the

Basic assumption: A probability space $(\tilde{\Omega},\tilde{\mathcal{B}},\tilde{P})$ is given and $\mathcal{B}$ and $\mathcal{C}$ are sub-σ-algebras of $\tilde{\mathcal{B}}$. The real random variable G is $\tilde{\mathcal{B}}$-measurable and the random variable W* is $\mathcal{C}$-measurable. Assuming

$$E|G| < \infty \tag{1}$$