Lecture IX. SUMS OF INDEPENDENT IDENTICALLY DISTRIBUTED RANDOM VARIABLES

Here I shall give an essentially self-contained derivation of the Berry-Esseen Theorem for sums of independently identically distributed random variables. Some of the analytic results of the second lecture will be used.

In order to establish the framework for a basic lemma, let $X_1,\ldots,X_{n+1}$ be independent random variables with common c.d.f. μ such that

$$EX_i = \int_{-\infty}^{\infty} t\,d\mu(t) = 0 \tag{1}$$

and

$$EX_i^2 = \int_{-\infty}^{\infty} t^2 d\mu(t) = \frac{1}{n}. \tag{2}$$

Also let

$$K(t) = n\int_t^{\infty} u\,d\mu(u) = -n\int_{-\infty}^{t} u\,d\mu(u). \tag{3}$$

Then K is a probability density function. The positivity of K follows from the first form of (3) when t is positive and from the second form when t is negative. The integral of K is easily seen to be 1:

$$\int_{-\infty}^{\infty} K(t)dt = \int_{-\infty}^{0} K(t)dt + \int_0^{\infty} K(t)dt \tag{4}$$